加法定理１の解答

氏名 ＿＿＿＿＿＿＿＿＿＿＿＿＿＿

1. 加法定理を用いて、次の三角比の値を求めよ。

(1) $\sin(-15^\circ)$

$= \sin(30^\circ - 45^\circ)$

$= \sin 30^\circ \cos 45^\circ - \cos 30^\circ \sin 45^\circ$

$= \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{2}}{2}$

答. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

(2) $\sin 105^\circ$

$= \sin(45^\circ + 60^\circ)$

$= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$

$= \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2}$

答. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$

(3) $\cos 15^\circ$

$= \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2}$

答. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(4) $\cos 75^\circ$

$= \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2}$

答. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(5) $\tan(-15^\circ)$

$= \tan(30^\circ - 45^\circ)$

$= \dfrac{\tan 30^\circ - \tan 45^\circ}{1 + \tan 30^\circ \tan 45^\circ} = \dfrac{\dfrac{\sqrt{3}}{3} - 1}{1 + \dfrac{\sqrt{3}}{3} \times 1}$

答. $-2+\sqrt{3}$

(6) $\tan 75^\circ$

$= \tan(30^\circ + 45^\circ)$

$= \dfrac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \dfrac{\dfrac{\sqrt{3}}{3} + 1}{1 - \dfrac{\sqrt{3}}{3} \times 1}$

答. $2+\sqrt{3}$

(7) $\cos(-75^\circ)$

$= \cos(45^\circ - 120^\circ)$

$= \cos 45^\circ \cos 120^\circ + \sin 45^\circ \sin 120^\circ$

$= \left(\dfrac{\sqrt{2}}{2}\right) \times \left(-\dfrac{1}{2}\right) + \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2}$

答. $\dfrac{-\sqrt{2}+\sqrt{6}}{4}$

(8) $\sin 15^\circ$

$= \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2}$

答. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(9) $\cos 165^\circ$

$= \cos(30^\circ + 135^\circ)$

$= \cos 30^\circ \cos 135^\circ - \sin 30^\circ \sin 135^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right) \times \left(-\dfrac{\sqrt{2}}{2}\right) - \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2}$

答. $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$

(10) $\tan 165^\circ$

$= \tan(30^\circ + 135^\circ)$

$= \dfrac{\tan 30^\circ + \tan 135^\circ}{1 - \tan 30^\circ \tan 135^\circ} = \dfrac{\dfrac{\sqrt{3}}{3} + (-1)}{1 - \dfrac{\sqrt{3}}{3} \times (-1)}$

答. $-2+\sqrt{3}$

加法定理２の解答

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1. 加法定理を用いて、次の三角比の値を求めよ。

（1） $\sin 75^\circ$

$= \sin(30^\circ + 45^\circ)$

$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

$= \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{2}}{2}$

答．$\dfrac{\sqrt{2} + \sqrt{6}}{4}$

（2） $\sin 165^\circ$

$= \sin(30^\circ + 135^\circ)$

$= \sin 30^\circ \cos 135^\circ + \cos 30^\circ \sin 135^\circ$

$= \dfrac{1}{2} \times \left(-\dfrac{\sqrt{2}}{2}\right) + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{2}}{2}$

答．$\dfrac{-\sqrt{2} + \sqrt{6}}{4}$

（3） $\cos(-15^\circ)$

$= \cos(30^\circ - 45^\circ)$

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$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2}$

答．$\dfrac{\sqrt{6} + \sqrt{2}}{4}$

（4） $\cos(-105^\circ)$

$= \cos(30^\circ - 135^\circ)$

$= \cos 30^\circ \cos 135^\circ + \sin 30^\circ \sin 135^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right) \times \left(-\dfrac{\sqrt{2}}{2}\right) + \dfrac{1}{2} \times \dfrac{\sqrt{2}}{2}$

答．$\dfrac{-\sqrt{6} + \sqrt{2}}{4}$

（5） $\tan 15^\circ$

$= \tan(45^\circ - 30^\circ)$

$= \dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \dfrac{1 - \dfrac{\sqrt{3}}{3}}{1 + 1 \times \dfrac{\sqrt{3}}{3}}$

答．$2 - \sqrt{3}$

（6） $\tan 75^\circ$

$= \tan(30^\circ + 45^\circ)$

$= \dfrac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \dfrac{\dfrac{\sqrt{3}}{3} + 1}{1 - \dfrac{\sqrt{3}}{3} \times 1}$

答．$2 + \sqrt{3}$

（7） $\sin(-75^\circ)$

$= \sin(45^\circ - 120^\circ)$

$= \sin 45^\circ \cos 120^\circ - \cos 45^\circ \sin 120^\circ$

$= \dfrac{\sqrt{2}}{2} \times \left(-\dfrac{1}{2}\right) - \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2}$

答．$\dfrac{-\sqrt{2} - \sqrt{6}}{4}$

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答．$\dfrac{-\sqrt{2} + \sqrt{6}}{4}$

（9） $\sin 195^\circ$

$= \sin(45^\circ + 150^\circ)$

$= \sin 45^\circ \cos 150^\circ + \cos 45^\circ \sin 150^\circ$

$= \dfrac{\sqrt{2}}{2} \times \left(-\dfrac{\sqrt{3}}{2}\right) + \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2}$

答．$\dfrac{-\sqrt{6} + \sqrt{2}}{4}$

（10） $\cos 105^\circ$

$= \cos(45^\circ + 60^\circ)$

$= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$

$= \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2} - \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2}$

答．$\dfrac{\sqrt{2} - \sqrt{6}}{4}$

加法定理３の解答

氏名 ＿＿＿＿＿＿＿＿＿＿＿＿＿＿

1. 三角関数の次の公式を導け。

（1）$\sin(\alpha+\beta)$

答．$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

（2）$\cos(\alpha+\beta)$

答．$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

（3）$\tan(\alpha+\beta)$

$=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

各項を $\cos\alpha\cos\beta$ で割って

$=\dfrac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$=\dfrac{\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{1}{1}+\dfrac{1}{1}\cdot\dfrac{\sin\beta}{\cos\beta}}{\dfrac{1}{1}\cdot\dfrac{1}{1}-\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}}$

$=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

答．$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

（4）$\tan(\alpha-\beta)$

$=\dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}=\dfrac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

各項を $\cos\alpha\cos\beta$ で割って

$=\dfrac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$=\dfrac{\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{1}{1}-\dfrac{1}{1}\cdot\dfrac{\sin\beta}{\cos\beta}}{\dfrac{1}{1}\cdot\dfrac{1}{1}+\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}}$

$=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

答．$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

（5）$\sin2\alpha$

$=\sin(\alpha+\alpha)$

$=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$

$=2\sin\alpha\cos\alpha$

答．$\sin2\alpha=2\sin\alpha\cos\alpha$

（6）$\tan2\alpha$

$=\tan(\alpha+\alpha)$

$=\dfrac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$

$=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

答．$\tan2\alpha=\dfrac{2\tan\alpha}{1-\tan^2\alpha}$

（7）$\cos^2\dfrac{\alpha}{2}$

2 倍角の公式 $\cos2\alpha=2\cos^2\alpha-1$

移項する

$2\cos^2\alpha=1+\cos2\alpha$　　両辺 2 で割る

$\cos^2\alpha=\dfrac{1+\cos2\alpha}{2}$

αを $\dfrac{\alpha}{2}$ に置き換えると

$\cos^2\dfrac{\alpha}{2}=\dfrac{1+\cos\alpha}{2}$

答．$\cos^2\dfrac{\alpha}{2}=\dfrac{1+\cos\alpha}{2}$

（8）$\tan^2\dfrac{\alpha}{2}$

$=\dfrac{\sin^2\dfrac{\alpha}{2}}{\cos^2\dfrac{\alpha}{2}}=\dfrac{\dfrac{1-\cos\alpha}{2}}{\dfrac{1+\cos\alpha}{2}}$

$=\dfrac{1-\cos\alpha}{1+\cos\alpha}$

答．$\tan^2\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{1+\cos\alpha}$

（9）$\sin\alpha\sin\beta$

$$\begin{array}{rcl}\cos(\alpha+\beta)&=&\cos\alpha\cos\beta-\sin\alpha\sin\beta\\ -\cos(\alpha-\beta)&=&\cos\alpha\cos\beta+\sin\alpha\sin\beta\\ \hline \cos(\alpha+\beta)-\cos(\alpha-\beta)&=&-2\sin\alpha\sin\beta\end{array}$$

両辺を 2 で割る

答．$\sin\alpha\sin\beta=-\dfrac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$

（10）$\cos\alpha\sin\beta$

$$\begin{array}{rcl}\sin(\alpha+\beta)&=&\sin\alpha\cos\beta+\cos\alpha\sin\beta\\ -\sin(\alpha-\beta)&=&\sin\alpha\cos\beta-\cos\alpha\sin\beta\\ \hline \sin(\alpha+\beta)-\sin(\alpha-\beta)&=&2\cos\alpha\sin\beta\end{array}$$

両辺を 2 で割る

答．$\cos\alpha\sin\beta=\dfrac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}$

加法定理４の解答

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1. 三角関数の次の公式を導け。

(1) $\cos(\alpha+\beta)$

答．$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(2) $\sin(\alpha+\beta)$

答．$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

(3) $\tan^2\frac{\alpha}{2}$

$=\dfrac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}=\dfrac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}}$

$=\dfrac{1-\cos\alpha}{1+\cos\alpha}$

答．$\tan^2\frac{\alpha}{2}=\dfrac{1-\cos\alpha}{1+\cos\alpha}$

(4) $\sin 2\alpha$

$=\sin(\alpha+\alpha)$

$=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$

$=2\sin\alpha\cos\alpha$

答．$\sin 2\alpha=2\sin\alpha\cos\alpha$

(5) $\sin^2\frac{\alpha}{2}$

2 倍角の公式 $\cos 2\alpha=1-2\sin^2\alpha$

移項する

$2\sin^2\alpha=1-\cos 2\alpha$　両辺 2 で割る

$\sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}$

αを$\frac{\alpha}{2}$に置き換えると

$\sin^2\frac{\alpha}{2}=\dfrac{1-\cos\alpha}{2}$

答．$\sin^2\frac{\alpha}{2}=\dfrac{1-\cos\alpha}{2}$

(6) $\sin A+\sin B$

$\alpha+\beta=A,\ \alpha-\beta=B$ とおくと

$\alpha=\dfrac{A+B}{2},\ \beta=\dfrac{A-B}{2}$ となる。

これを下記の式に代入する。

$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$

$\sin A+\sin B=2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

答．$\sin A+\sin B=2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

(7) $\tan(\alpha+\beta)$

$=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

各項を $\cos\alpha\cos\beta$　で割って

$=\dfrac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$=\dfrac{\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{1}{1}+\dfrac{1}{1}\cdot\dfrac{\sin\beta}{\cos\beta}}{\dfrac{1}{1}\cdot\dfrac{1}{1}-\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}}$

$=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

答．$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(8) $\cos(\alpha+\beta)$

答．$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

(9) $\sin(\alpha-\beta)$

答．$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

(10) $\cos 3\alpha$

$\cos(2\alpha+\alpha)=\cos 2\alpha\cos\alpha-\sin 2\alpha\sin\alpha$

$=(2\cos^2\alpha-1)\cdot\cos\alpha-2\sin\alpha\cos\alpha\cdot\sin\alpha$

$=2\cos^3\alpha-\cos\alpha-2(1-\cos^2\alpha)\cdot\cos\alpha$

$=2\cos^3\alpha-\cos\alpha-2\cos\alpha+2\cos^3\alpha$

$=4\cos^3\alpha-3\cos\alpha$

高三のヨーコ参上まだ未婚

答．$\cos 3\alpha=4\cos^3\alpha-3\cos\alpha$