

加法定理1の解答

氏名 _____

1. 加法定理を用いて、次の三角比の値を求めよ。

$$\begin{aligned} (1) \sin(-15^\circ) &= \sin(30^\circ - 45^\circ) \\ &= \sin 30^\circ \cos 45^\circ - \cos 30^\circ \sin 45^\circ \\ &= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{\sqrt{2} - \sqrt{6}}{4}$

$$\begin{aligned} (2) \sin 105^\circ &= \sin(45^\circ + 60^\circ) \\ &= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ \\ &= \frac{\sqrt{2}}{2} \times \frac{1}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \end{aligned}$$

答. $\frac{\sqrt{2} + \sqrt{6}}{4}$

$$\begin{aligned} (3) \cos 15^\circ &= \cos(45^\circ - 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} \end{aligned}$$

答. $\frac{\sqrt{6} + \sqrt{2}}{4}$

$$\begin{aligned} (4) \cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{\sqrt{6} - \sqrt{2}}{4}$

$$\begin{aligned} (5) \tan(-15^\circ) &= \tan(30^\circ - 45^\circ) \\ &= \frac{\tan 30^\circ - \tan 45^\circ}{1 + \tan 30^\circ \tan 45^\circ} = \frac{\frac{\sqrt{3}}{3} - 1}{1 + \frac{\sqrt{3}}{3} \times 1} \end{aligned}$$

答. $-2 + \sqrt{3}$

$$\begin{aligned} (6) \tan 75^\circ &= \tan(30^\circ + 45^\circ) \\ &= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \times 1} \end{aligned}$$

答. $2 + \sqrt{3}$

$$\begin{aligned} (7) \cos(-75^\circ) &= \cos(45^\circ - 120^\circ) \\ &= \cos 45^\circ \cos 120^\circ + \sin 45^\circ \sin 120^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right) \times \left(-\frac{1}{2}\right) + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \end{aligned}$$

答. $\frac{-\sqrt{2} + \sqrt{6}}{4}$

$$\begin{aligned} (8) \sin 15^\circ &= \sin(45^\circ - 30^\circ) \\ &= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} \end{aligned}$$

答. $\frac{\sqrt{6} - \sqrt{2}}{4}$

$$\begin{aligned} (9) \cos 165^\circ &= \cos(30^\circ + 135^\circ) \\ &= \cos 30^\circ \cos 135^\circ - \sin 30^\circ \sin 135^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right) \times \left(-\frac{\sqrt{2}}{2}\right) - \frac{1}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{-\sqrt{6} - \sqrt{2}}{4}$

$$\begin{aligned} (10) \tan 165^\circ &= \tan(30^\circ + 135^\circ) \\ &= \frac{\tan 30^\circ + \tan 135^\circ}{1 - \tan 30^\circ \tan 135^\circ} = \frac{\frac{\sqrt{3}}{3} + (-1)}{1 - \frac{\sqrt{3}}{3} \times (-1)} \end{aligned}$$

答. $-2 + \sqrt{3}$

加法定理2の解答

氏名 _____

1. 加法定理を用いて、次の三角比の値を求めよ。

(1) $\sin 75^\circ$

$$\begin{aligned} &= \sin(30^\circ + 45^\circ) \\ &= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ \\ &= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{\sqrt{2} + \sqrt{6}}{4}$

(2) $\sin 165^\circ$

$$\begin{aligned} &= \sin(30^\circ + 135^\circ) \\ &= \sin 30^\circ \cos 135^\circ + \cos 30^\circ \sin 135^\circ \\ &= \frac{1}{2} \times \left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{-\sqrt{2} + \sqrt{6}}{4}$

(3) $\cos(-15^\circ)$

$$\begin{aligned} &= \cos(30^\circ - 45^\circ) \\ &= \cos 30^\circ \cos 45^\circ + \sin 30^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{\sqrt{6} + \sqrt{2}}{4}$

(4) $\cos(-105^\circ)$

$$\begin{aligned} &= \cos(30^\circ - 135^\circ) \\ &= \cos 30^\circ \cos 135^\circ + \sin 30^\circ \sin 135^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right) \times \left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2} \times \frac{\sqrt{2}}{2} \end{aligned}$$

答. $\frac{-\sqrt{6} + \sqrt{2}}{4}$

(5) $\tan 15^\circ$

$$\begin{aligned} &= \tan(45^\circ - 30^\circ) \\ &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \times \frac{\sqrt{3}}{3}} \end{aligned}$$

答. $2 - \sqrt{3}$

(6) $\tan 75^\circ$

$$\begin{aligned} &= \tan(30^\circ + 45^\circ) \\ &= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \times 1} \end{aligned}$$

答. $2 + \sqrt{3}$

(7) $\sin(-75^\circ)$

$$\begin{aligned} &= \sin(45^\circ - 120^\circ) \\ &= \sin 45^\circ \cos 120^\circ - \cos 45^\circ \sin 120^\circ \\ &= \frac{\sqrt{2}}{2} \times \left(-\frac{1}{2}\right) - \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \end{aligned}$$

答. $\frac{-\sqrt{2} - \sqrt{6}}{4}$

(8) $\cos(-75^\circ)$

$$\begin{aligned} &= \cos(45^\circ - 120^\circ) \\ &= \cos 45^\circ \cos 120^\circ + \sin 45^\circ \sin 120^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right) \times \left(-\frac{1}{2}\right) + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \end{aligned}$$

答. $\frac{-\sqrt{2} + \sqrt{6}}{4}$

(9) $\sin 195^\circ$

$$\begin{aligned} &= \sin(45^\circ + 150^\circ) \\ &= \sin 45^\circ \cos 150^\circ + \cos 45^\circ \sin 150^\circ \\ &= \frac{\sqrt{2}}{2} \times \left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2} \times \frac{1}{2} \end{aligned}$$

答. $\frac{-\sqrt{6} + \sqrt{2}}{4}$

(10) $\cos 105^\circ$

$$\begin{aligned} &= \cos(45^\circ + 60^\circ) \\ &= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ \\ &= \frac{\sqrt{2}}{2} \times \frac{1}{2} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \end{aligned}$$

答. $\frac{\sqrt{2} - \sqrt{6}}{4}$

加法定理3の解答

氏名 _____

1. 三角関数の次の公式を導け。

(1) $\sin(\alpha + \beta)$

答. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

(2) $\cos(\alpha + \beta)$

答. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

(3) $\tan(\alpha + \beta)$

$$\begin{aligned} &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\ &\text{各項を } \cos \alpha \cos \beta \text{ で割って} \\ &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\frac{\sin \alpha}{\cos \alpha} \cdot \frac{1}{1} + \frac{1}{1} \cdot \frac{\sin \beta}{\cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\frac{1}{1} \cdot \frac{1}{1} + \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \end{aligned}$$

答. $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

(4) $\tan(\alpha - \beta)$

$$\begin{aligned} &= \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta} \\ &\text{各項を } \cos \alpha \cos \beta \text{ で割って} \\ &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\frac{\sin \alpha}{\cos \alpha} \cdot \frac{1}{1} - \frac{1}{1} \cdot \frac{\sin \beta}{\cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\frac{1}{1} \cdot \frac{1}{1} - \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \end{aligned}$$

答. $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

(5) $\sin 2\alpha$

$$\begin{aligned} &= \sin(\alpha + \alpha) \\ &= \sin \alpha \cos \alpha + \cos \alpha \sin \alpha \\ &= 2 \sin \alpha \cos \alpha \end{aligned}$$

答. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

(6) $\tan 2\alpha$

$$\begin{aligned} &= \tan(\alpha + \alpha) \\ &= \frac{\tan \alpha + \tan \alpha}{1 - \tan \alpha \tan \alpha} \\ &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \end{aligned}$$

答. $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$

(7) $\cos^2 \frac{\alpha}{2}$

2倍角の公式 $\cos 2\alpha = 2 \cos^2 \alpha - 1$

移項する

$2 \cos^2 \alpha = 1 + \cos 2\alpha$ 両辺2で割る

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

α を $\frac{\alpha}{2}$ に置き換えると

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

答. $\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$

(8) $\tan^2 \frac{\alpha}{2}$

$$\begin{aligned} &= \frac{\sin^2 \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2}} = \frac{1 - \cos \alpha}{1 + \cos \alpha} \\ &= \frac{1 - \cos \alpha}{1 + \cos \alpha} \end{aligned}$$

答. $\tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$

(9) $\sin \alpha \sin \beta$

$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ -\cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \hline \cos(\alpha + \beta) - \cos(\alpha - \beta) &= -2 \sin \alpha \sin \beta \end{aligned}$$

両辺を2で割る

答. $\sin \alpha \sin \beta = -\frac{1}{2} \{\cos(\alpha + \beta) - \cos(\alpha - \beta)\}$

(10) $\cos \alpha \sin \beta$

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ -\sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \hline \sin(\alpha + \beta) - \sin(\alpha - \beta) &= 2 \cos \alpha \sin \beta \end{aligned}$$

両辺を2で割る

答. $\cos \alpha \sin \beta = \frac{1}{2} \{\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$

加法定理4の解答

氏名 _____

1. 三角関数の次の公式を導け。

(1) $\cos(\alpha + \beta)$

答. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

(2) $\sin(\alpha + \beta)$

答. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

(3) $\tan^2 \frac{\alpha}{2}$

$$= \frac{\sin^2 \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2}} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

$$= \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

答. $\tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$

(4) $\sin 2\alpha$

$$= \sin(\alpha + \alpha)$$

$$= \sin \alpha \cos \alpha + \cos \alpha \sin \alpha$$

$$= 2 \sin \alpha \cos \alpha$$

答. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

(5) $\sin^2 \frac{\alpha}{2}$
 2倍角の公式 $\cos 2\alpha = 1 - 2 \sin^2 \alpha$
 移項する

$$2 \sin^2 \alpha = 1 - \cos 2\alpha \quad \text{両辺2で割る}$$

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$
 α を $\frac{\alpha}{2}$ に置き換えると

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

答. $\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$

(6) $\sin A + \sin B$

$\alpha + \beta = A, \alpha - \beta = B$ とおくと
 $\alpha = \frac{A+B}{2}, \beta = \frac{A-B}{2}$ となる。
 これを下記の式に代入する。

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

答. $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

(7) $\tan(\alpha + \beta)$

$$= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}$$
 各項を $\cos \alpha \cos \beta$ で割って

$$= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha} \cdot \frac{1}{1} + \frac{1}{1} \cdot \frac{\sin \beta}{\cos \beta}}{\frac{1}{1} \cdot \frac{1}{1} - \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}}$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

答. $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

(8) $\cos(\alpha + \beta)$

答. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

(9) $\sin(\alpha - \beta)$

答. $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

(10) $\cos 3\alpha$

$$\cos(2\alpha + \alpha) = \cos 2\alpha \cos \alpha - \sin 2\alpha \sin \alpha$$

$$= (2 \cos^2 \alpha - 1) \cdot \cos \alpha - 2 \sin \alpha \cos \alpha \cdot \sin \alpha$$

$$= 2 \cos^3 \alpha - \cos \alpha - 2(1 - \cos^2 \alpha) \cdot \cos \alpha$$

$$= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha$$

$$= 4 \cos^3 \alpha - 3 \cos \alpha$$

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答. $\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$